Letters

Further Comments on "Analysis of Wide Inclined Slot Coupled Narrow Wall Coupler Between Dissimilar Rectangular Waveguides"

Sembiam R. Rengarajan

The reply to our recent comments on the above paper[1] has not addressed the main issue raised in [2]. D. Satyanarayana and A. Chakrabarty apparently do not understand the significance of the incompleteness of the eigenfunction expansions for the dyadic Green's functions in the source region of waveguides, and the relationship between the source region singularity, and the 'TE$_{00}$ mode'. This conclusion is based on 1) their statement, "Also, it is no way connected with the singularity as pointed out in the comments" [1], and 2) by the fact that they do not understand the discussions on the singularity term containing $\delta(z-z')$ in our original paper and still they seem to be not clear whether our Green's functions are complete [3]. Therefore additional discussions are warranted.

The incompleteness of the eigenfunction expansions of the dyadic Green's functions in the source region is well known [4]. Discussions on the source region singularity of the dyadic Green's functions and considerations on working with such a singularity have been addressed in the literature (see, for example, [5]–[8]). Therefore we shall discuss the specific relationship between the singularity in the Green's functions and the 'TE$_{00}$ mode'.

Let us define the dyadic Green's function $\overline{\overline{G}}(\boldsymbol{R}, \boldsymbol{R}')$ for the magnetic field due to a magnetic current inside a rectangular waveguide, with a time dependence assumed in the form $\exp(j\omega t)$, as follows

$$\boldsymbol{H}(\boldsymbol{R}) = \overline{\overline{G}}(\boldsymbol{R}, \boldsymbol{R}') \cdot \boldsymbol{M}(\boldsymbol{R}')\, ds'. \tag{1}$$

Here the magnetic current $\boldsymbol{M}(\boldsymbol{R}')$ is $\boldsymbol{E} \times \hat{n}$ where $\hat{n}$ is the unit normal vector directed into the waveguide, and $\boldsymbol{E}$ is the aperture electric field of the slot. $\boldsymbol{R}$ and $\boldsymbol{R}'$ are position vectors corresponding to a source point and a field point, respectively, and the integration is carried out over the aperture region.

The dyadic Green's function $\overline{\overline{G}}(\boldsymbol{R}, \boldsymbol{R}')$ in (1) can be expressed in terms of two parts, $\overline{\overline{G}}_s$ and $\overline{\overline{G}}_{ns}$. The singular part of the dyadic Green's function $\overline{\overline{G}}_s$ is zero for $\boldsymbol{R} \neq \boldsymbol{R}'$ and $\overline{\overline{G}}_{ns}$ is the non-singular term. Clearly the second term consists only of the eigenfunctions of the waveguide and there is no contribution from the 'TE$_{00}$ mode' to this term. The singular term is given by (see, e.g., [9])

$$\overline{\overline{G}}_s(\boldsymbol{R}, \boldsymbol{R}') = \hat{z}\hat{z}\frac{j}{\omega\mu_0}\delta(\boldsymbol{R} - \boldsymbol{R}'). \tag{2}$$

Here $\hat{z}$ is a unit vector along the wave guide axis, ω is the angular frequency, μ_0 is the permeability and $j^2 = -1$.

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The author is with the Department of Electrical and Computer Engineering, California State University, Northridge, CA 91330-8346 USA.
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[1] D. Satyanarayana and A. Chakraborty, *IEEE Trans. Microwave Theory Tech.*, vol. 42, no. 5, pp. 914–917, May 1994.

Note that $\delta(\boldsymbol{R}-\boldsymbol{R}') = \delta(x-x')\delta(y-y')\delta(z-z')$ and that the first two delta functions may be expressed in series form as given below

$$\delta(x-x') = \frac{2}{a}\sum_{m=0}^{\infty}\epsilon_m^2 \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{m\pi x'}{a}\right)$$

where $\epsilon_m^2 = 1$ if $m \neq 0$ and $\epsilon_m^2 = 1/2$ if $m = 0$.

Similarly, $\delta(y-y') = 2/b\sum_{n=0}^{\infty}\epsilon_n^2 \cos(n\pi y/b)\cos(n\pi y'/b)$.

Therefore, the singular part of the dyadic Green's function is given by

$$\overline{\overline{G}}_s(\boldsymbol{R}, \boldsymbol{R}') = \hat{z}\hat{z}\frac{j}{\omega\mu_0}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{4\epsilon_{mn}^2}{ab}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{m\pi x'}{a}\right) \cdot\cos\left(\frac{n\pi y}{b}\right)\cos\left(\frac{n\pi y'}{b}\right)\delta(z-z'). \tag{3}$$

In (3) the waveguide inner dimensions are a and b and $\epsilon_{mn}^2 = \epsilon_m^2\epsilon_n^2$. Thus we note that the singular term of the dyadic Green's functions contains a double infinite series. The terms corresponding to $m = 0$ and $n = 0$ in this series, interpreted as TE$_{00}$ mode by D. Satyanarayana and A. Chakrabarty, as well as all the terms in the series are nonpropagating and hence strictly they cannot be interpreted as TE$_{00}$ mode and waveguide eigenmodes, respectively. These terms merely express the source region singularity. Since there is a discussion on the singularity term containing the Dirac delta term in our paper [3], one wonders how D. Satyanarayana and A. Chakrabarty, without understanding the significance of the singularity would simply conclude that the '00' term is missing in our work [10]. Obviously, all significant terms in (3), including the '00' term as well as the nonsingular part of the Green's functions have to be properly evaluated for accurate computation of fields.

In [1], the authors state that, "It is not clear from the paper [2] whether the 'TE$_{00}$ mode' was considered or not." Even though they are *still* not clear about it, they already concluded in [10] that our paper did not consider the 'TE$_{00}$ mode'. Also it is strange that instead of accepting their mistake, they quote some other authors stating that the '00' mode cannot be ignored, as though we advocated ignoring such a term.

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Authors' Reply by D. Satyanarayana and Ajay Chakraborty

The authors would like to thank S. R. Rengarajan for his further comments on our recent paper,[2] which has helped the authors to obtain an insight into the dyadic Green's function for the magnetic field used by him.

Rengarajan [1] has clarified the doubts raised by the authors, primarily through the missing steps which probably could not be accommodated in his papers because of lack of space. However, the magnetic field expression derived by the authors need not go through all the complicacies which Rengarajan has gone through by referring to the literature [2]–[6]. The end result obtained by the authors is now found to be exactly the same with that obtained using the expression given by Rengarajan and these are described below. The confusion arose because in his publication neither the in between steps nor the expression of the end results were given to make them useful for microwave community. The steps of the magnetic field expression derived by the authors is given below.

The z-directed internal scattered magnetic field in a rectangular waveguide is given by the expression

$$\overline{H}(y, z) = -\frac{j}{\omega\mu}(k^2 + \nabla\nabla) \cdot \int\int \overline{E}' \times \hat{n} \cdot G(r/r')\, ds'$$

For the simple case of axial slot positioned with a lateral displacement of y_o from one end on the broad wall

$$\overline{E}' = \hat{U}_y \cos\frac{\pi z'}{2L} \qquad \begin{cases} |z'| \le L \\ |y'| \le W \end{cases},$$

$$\hat{n} = -\hat{U}_x$$

and the $G(r/r')$ is the Green's functions pertaining to the internal scattered field of a rectangular waveguide given by (for the z-directed scattered field)

$$G(r/r') = \hat{U}_z\hat{U}_z \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{\epsilon_m \epsilon_n}{2\gamma ab} \cos\frac{m\pi x}{a} \cdot \cos\frac{m\pi x'}{a} \cos\frac{n\pi y}{b} \cos\frac{n\pi y'}{b} e^{-\gamma|z-z'|}$$

Upon performing the double integration over slot aperture (i.e., over y' and z' and at x'=0), H_z is obtained as

$$H_z(y, z) = \frac{j}{\omega\mu} \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{\epsilon_m \epsilon_n}{2\gamma ab} \cos\frac{m\pi x}{a} \cdot \cos\frac{n\pi y}{b} \left[\frac{2\cos\frac{n\pi y_o}{b} \sin\frac{n\pi W}{b}}{\frac{n\pi}{b}}\right] \cdot \left(k^2 + \frac{\partial^2}{\partial z^2}\right)\left[2\gamma \cos\frac{\pi z}{2L} + \frac{2\pi}{2L} e^{\gamma L} \sinh\gamma z\right]$$

$$= \frac{j}{\omega\mu} \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} \frac{\epsilon_m \epsilon_n}{2\gamma ab} \cos\frac{m\pi x}{a} \cdot \cos\frac{n\pi y}{b} \left[\frac{2\cos\frac{n\pi y_o}{b} \sin\frac{n\pi W}{b}}{\frac{n\pi}{b}}\right] \cdot \left[2\gamma\left(k^2 - \frac{\pi}{2L}\right)^2 \cos\frac{\pi z}{2L} + \frac{2\pi}{2L} e^{-\gamma L}(k^2 + \gamma^2)\sinh\gamma z\right].$$

This expression is identical to that of Rengarajan's as given in his further comments.

It appears in some of the publications that the objective is to keep the reader in the dark about the details of the procedure. Whatever comments are made by the authors were a representative feeling of a broad section of the microwave community.

However, regarding the paper [7] the authors would like to stress that it is definitely a step forward toward the analysis of a wide inclined slot in a waveguide using the entire domain basis and same testing functions. It may be pointed out that it has not been possible to use testing functions with dirac delta in the direction of the width because that equals narrow width approximations.

The authors fully agree that many more steps are to be covered for a complete analysis, namely:

1) Inclusion of transverse component of equivalent magnetic current in the slot aperture.
2) Inclusion of correct transverse variation of respective slot aperture field components.
3) Enforcing the boundary conditions for the transverse magnetic field components.

But that involves increasing the amount of computation with the same technique, and anybody with such requirement can perform this analysis [7] using a large number of basis functions in both directions and additional computer time. However, that is a simple extension of the simplified method of whatever is suggested by the author. The authors feel that these clarifications are sufficient for this series of arguments.

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The authors are with the Department of Electronics and Electrical Communication Engineering, Indian Institute of Technology, Kharagpur 721 302, India.

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